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On the relationship between time and frequency domain methods in time delay estimation for leak detection in water distribution pipes

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Abstract

Estimation of time delay in a vibroacoustic system is a problem that occurs in several engineering fields. This can be determined directly in the time domain or in the frequency domain by examining the phase spectrum of two signals. In this paper, the equivalence of these two methods is discussed with particular reference to the problem of determining the position of a leak in water distribution pipes. Popular methods central to this process are based on cross-correlation, and it has been found that pre-whitening the signals prior to determining the cross-correlation function has certain advantages. A new interpretation of the process of cross-correlation for time delay estimation is presented. To support the theoretical findings, analysis is carried out on test data from a specially constructed leak-detection facility located at a National Research Council site in Canada. Test results show that the time delay estimates and their variances calculated using time and frequency domain methods are almost identical.

1. Introduction

Time delay estimation in vibroacoustic systems is of interest in many engineering applications [1]. This can be done directly in the time domain or in the frequency domain by examining the phase spectrum of two signals. Popular methods central to the process of estimating time delay are based on cross-correlation methods, including the basic cross-correlation (BCC) method and generalised cross-correlation (GCC) methods [2–4]. The essential difference between the BCC and the GCC methods is that with the latter, the signals are passed though filters prior to performing the cross-correlation. Although time delay can be calculated directly in the time domain or indirectly in the frequency domain using these methods, they essentially determine the time delay corresponding to the maximum value of the cross-correlation function.

The performance of various correlation-based time delay estimators has been discussed by Knapp and Carter [2], and by the authors for the purposes of leak detection in buried plastic water pipes [5]. It was shown

Abbreviations: BCC, basic cross-correlation function; COH, coherence; GCC, generalised cross-correlation function; GPS, generalised phase spectrum; ML, maximum likelihood; PHAT, phase transform; SCOT, smoothed coherence transform.

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Nomenclature

c	speed of leak noise propagation
d	distance between the two sensors
d_1, d_2	distance between the leak and sensors 1 and 2
e^2	mean square error
N	data points per segment
r	number of segments in the time series
$R^g_{x_1x_2}(\tau)$, $\hat{R}^g_{x_1x_2}(\tau)$	GCC function and its estimate
$S_{x_1x_2}(\omega)$, $\hat{S}_{x_1x_2}(\omega)$	CSD function and its estimate
$S_{x_1x_1}(\omega)$, $\hat{S}_{x_1x_1}(\omega)$	ASD function of $x_1(t)$ and its estimate
$S_{x_2x_2}(\omega)$, $\hat{S}_{x_2x_2}(\omega)$	ASD function of $x_2(t)$ and its estimate
T_r, T	time duration per segment and whole observation time
$W_g(\omega)$	frequency weighting function of the GPS method
$x_1(t)$, $x_2(t)$	acoustic/vibration signals
$\gamma^2_{x_1x_2}(\omega)$, $\hat{\gamma}^2_{x_1x_2}(\omega)$	coherence function and its estimate
$\sigma^2_{\hat{\tau}\,\mathrm{peak}}$, $\sigma_{\hat{\tau}\,\mathrm{peak}}$	variance and standard derivation of $\hat{\tau}_{\mathrm{peak}}$
τ, τ_{peak}, $\hat{\tau}_{\mathrm{peak}}$	lag of time; time delay at the peak value and its estimate
σ_z	standard deviation of the first derivative of the GCC function
$\sigma^2_{\hat{\Phi}_i}$	variance of $\hat{\Phi}_i$
$\Phi_{x_1x_2}(\omega)$, $\hat{\Phi}_{x_1x_2}(\omega)$	phase spectrum and its estimate
$\Psi_g(\omega)$	frequency weighting function of the GCC function
$\delta\omega$	frequency increment
$\Delta\omega$, ω_0, ω_1	frequency bandwidth of band-pass filter; lower and upper cut-off frequencies of band-pass filter
ω_i, W_i, $\hat{\Phi}_i$, $\hat{S}_i$, $\hat{\gamma}^2_i$	discrete form of ω, $W_g(\omega)$, $\hat{\Phi}_{x_1x_2}(\omega)$, $\hat{S}_{x_1x_2}(\omega)$, $\hat{\gamma}^2_{x_1x_2}(\omega)$ evaluated at the ith frequency

in the latter paper that the phase transform (PHAT), smoothed coherence transform (SCOT), and maximum likelihood (ML) GCC methods, that pre-whiten the leak signals prior to the cross-correlation, have the desirable feature of sharpening the peak in the cross-correlation function. A brief description of the GCC methods considered here is given in the Appendix.

An alternative method of determining the time delay between two signals is to calculate the gradient of the phase of their cross-spectral density (CSD) with respect to frequency. Piersol [6] showed that regression analysis of the phase spectrum at selected frequencies yields time delay estimates with the same accuracy as the PHAT method. However, strict control on the selection of frequencies used in the calculation of the time delay estimate is required. Using a frequency dependent weighting function, Zhao and Hou [7] developed a new generalised phase spectrum (GPS) method. Rather than weighting the modulus of the CSD, however, the GPS method weights the phase spectrum. Each GCC processor has a corresponding GPS method.

The aim of this paper is to show explicitly the equivalence of the time delay estimators based on the GCC and the GPS methods, with particular reference to the problem of determining the position of a leak in water distribution pipes. The results of the analysis facilitate a new interpretation of the process of cross-correlation for time delay estimation, and this is also presented.

An alternative coherence weighted method (COH) is proposed as a result of the analysis between the GCC and the GPS methods. Compared to the phase gradient method [6], which is termed here the GPS-PHAT method, the GPS-COH method (coherence weighted phase spectrum) is potentially more accurate, because a coherence-based weighting function is used to suppress those frequency regions where the data is dominated by noise. To validate the equivalence between the time and frequency domain methods and to compare the performance of the GCC-COH (coherence weighted generalised cross-correlation method), BCC, GPS-COH and GPS-BCC methods, the time delay estimators are applied to experimental data from buried water pipes.

2. Overview of leak detection and the GCC and GPS methods

2.1. Leak detection in pipes

To determine the position of a leak in water distribution pipes, vibration or acoustic signals are measured at two access points using sensors such as accelerometers or hydrophones, either side of the location of a suspected leak, as shown in Fig. 1. If a leak exists, a distinct peak may be found in the cross-correlation of the two signals $x_1(t)$ and $x_2(t)$. This gives an estimate of the time delay τ_{peak} that corresponds to the difference in arrival times between the acoustic signals at each sensor. The location of the leak relative to one of the measurement points d_1, can be calculated using the relationship between the time delay τ_{peak}, the distance d between the access points, and the propagation speed of the leak noise c, in the buried pipe,

$$d_1 = \frac{d - c\tau_{\text{peak}}}{2}. \tag{1}$$

2.2. GCC methods

A comparison of the GCC methods was first presented by Knapp and Carter [2] and by the authors for those applicable to leak detection in plastic water pipes [5]. The time delay is determined by calculating the time at which the weighted CSD of the leak noise signals given by [1]

$$\hat{R}^g_{x_1x_2}(\tau) = F^{-1}\{\Psi_g(\omega)\hat{S}_{x_1x_2}(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Psi_g(\omega)\hat{S}_{x_1x_2}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\,\mathrm{d}\omega, \tag{2}$$

is a maximum, where $\hat{S}_{x_1x_2}(\omega)$ is the smooth estimate of the CSD function $S_{x_1x_2}(\omega)$ and $\Psi_g(\omega)$ is the weighting function, which is given in Table 1 for several of the common GCC methods. When $\Psi_g(\omega) = 1$, the GCC methods all reduce to the BCC method. GCC methods offer a potential improvement over the BCC method. An example of this is in the application of leak detection. For example, the analytical model of the cross-correlation function of leak signals in plastic pipes developed by Gao et al. [8,9] shows that a plastic pipe essentially acts as an acoustic low-pass filter, which affects the estimation of the time delay. The filtering properties of the pipe can, to some extent, be compensated for by pre-whitening the signals prior to calculating the cross-correlation function. For instance, the GCC-PHAT "flattens" the modulus of the CSD spectrum and thus effectively only transforms the phase of the CSD spectrum, between the leak signals measured at two locations, into the time domain cross-correlation function.

For signals where there is a single time delay the phase spectrum has a constant gradient, so the corresponding correlation function for signals with infinite bandwidth is a delta function. In practical situations, however, the estimate of the actual time delay is corrupted due to the existence of background noise and the filtering effect of the pipe on the phase spectrum. A typical measured phase spectrum from leak noise signals is shown in Fig. 2, together with an estimate of the actual phase spectrum. The leak noise signals are dominated by ambient noise at frequencies below ω_0 and are attenuated by the filtering properties of the pipe so that they are below the ambient noise floor at frequencies greater than ω_1. Using the GCC-SCOT and GCC-ML methods, the time delay estimate can be improved by weighting the signals at each frequency according to the coherence. However, a frequency band $\Delta\omega$ must be specified in the calculation procedure.

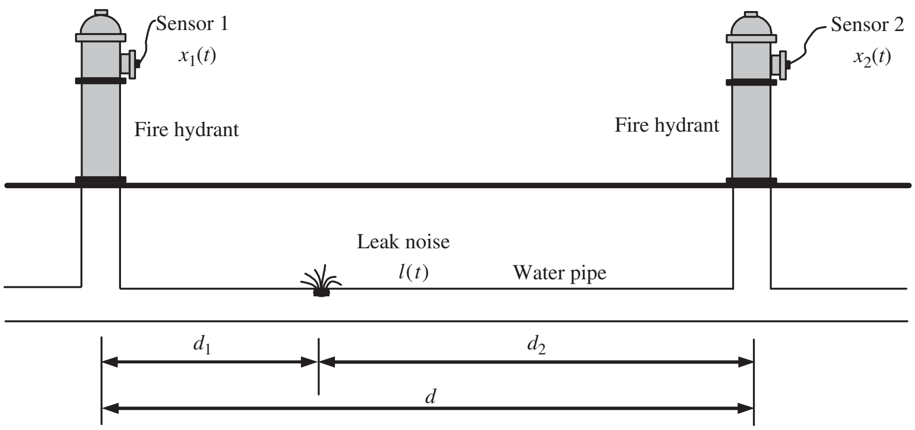

Fig. 1. Schematic of a pipe with a leak bracketed by two sensors.

Table 1
Weighting functions used in the correlation methods and the corresponding weighting functions in the GPS methods

Method	$\Psi_g(\omega)$ (GCC)	$W_g(\omega)$ (GPS)
BCC	1	$\lvert S_{x_1x_2}(\omega)\rvert$
PHAT	$1/\lvert S_{x_1x_2}(\omega)\rvert$	1
SCOT	$\gamma_{x_1x_2}(\omega)/\lvert S_{x_1x_2}(\omega)\rvert$	$\gamma_{x_1x_2}(\omega)$
ML	$\gamma^2_{x_1x_2}(\omega)/\{[1-\gamma^2_{x_1x_2}(\omega)]\lvert S_{x_1x_2}(\omega)\rvert$	$\gamma^2_{x_1x_2}(\omega)/[1-\gamma^2_{x_1x_2}(\omega)$
COH	$\gamma^2_{x_1x_2}(\omega)/\lvert S_{x_1x_2}(\omega)\rvert$	$\gamma^2_{x_1x_2}(\omega)$

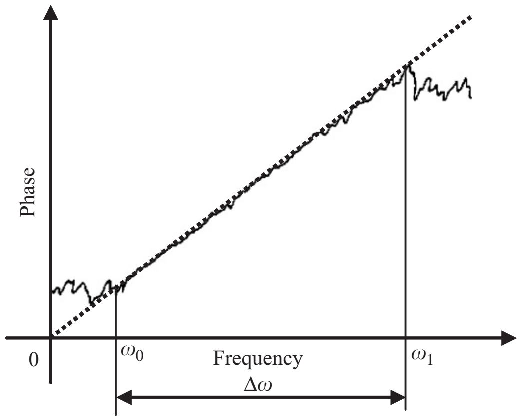

Fig. 2. Illustration of selection of the cut-off frequencies ω_0 and ω_1, and frequency bandwidth $\Delta\omega$ using the prewhitening GCC methods: measured phase (——); phase corresponding to the actual time delay (⋯⋯).

This has a detrimental effect in that the standard deviation and the resolution of the time delay estimate are a function of the frequency bandwidth $\Delta\omega$ [5].

2.3. The GPS method

In this section, the GPS method is outlined. The best time delay estimate between two sensor signals $\hat{\tau}_{\text{peak}}$ is defined as that which minimises the mean square error between the measured phase of the CSD and the estimate of the phase of the CSD corresponding to the time delay in a frequency band of interest. Consider a phase estimate $\hat{\Phi}_i$ at frequency ω_i computed from two sensor signals $x_1(t)$ and $x_2(t)$. Assuming that the total record observation time T is divided into r independent segments, with N data points per segment, the mean square error e^2 between the measured data and the linear phase estimate $\omega_i\hat{\tau}_{\text{peak}}$ is given by

$$e^2 = \sum_{i=1}^{N/2} W_i(\hat{\Phi}_i + \omega_i\hat{\tau}_{\text{peak}})^2, \tag{3}$$

where $i = 1,2,\ldots,N/2$; W_i is a frequency dependent weighting function at frequency ω_i. When e^2 is a minimum with respect to $\hat{\tau}$, then

$$\frac{\partial e^2}{\partial \hat{\tau}_{\text{peak}}} = 0 \tag{4}$$

Thus, differentiating the mean square error with respect to $\hat{\tau}_{\text{peak}}$, setting the resulting expression to zero and rearranging, gives the time delay estimate

$$\hat{\tau}_{\text{peak}} = -\frac{\sum_{i=1}^{N/2} W_i\hat{\Phi}_i\omega_i}{\sum_{i=1}^{N/2} W_i\omega_i^2}. \tag{5}$$

Eq. (5) gives the GPS method for time delay estimation [7]. The equivalence of time delay estimation using the GCC and GPS methods is discussed in the next section by comparing the time delay estimates and the variances determined using both methods.

3. Equivalence of the GCC and GPS methods

3.1. Comparison of time delay estimation

The weighted cross-correlation function $\hat{R}^g_{x_1x_2}(\tau)$ between two signals is given by Eq. (2). To compare the time delay estimates determined using the GCC and GPS methods, the correlation function given by Eq. (2) must be written in discrete form. Eq. (2) can first be rewritten as

$$\hat{R}^g_{x_1x_2}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} W_g(\omega)\mathrm{e}^{\mathrm{i}[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]}\,\mathrm{d}\omega, \tag{6}$$

where

$$W_g(\omega) = \Psi_g(\omega)|\hat{S}_{x_1x_2}(\omega)|. \tag{7}$$

Differentiating Eq. (6) with respect to τ gives

$$\frac{\partial \hat{R}^g_{x_1x_2}(\tau)}{\partial\tau} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{i}\omega W_g(\omega)\mathrm{e}^{\mathrm{i}[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]}\,\mathrm{d}\omega. \tag{8}$$

If it is assumed that $\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau \ll 1$, then $\mathrm{e}^{\mathrm{i}[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]} \approx 1+\mathrm{i}[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]$, which can be substituted into Eq. (8) to give

$$\frac{\partial \hat{R}^g_{x_1x_2}(\tau)}{\partial\tau} \approx \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{i}\omega W_g(\omega)\{1+\mathrm{i}[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]\}\,\mathrm{d}\omega. \tag{9}$$

Now, $\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{i}\omega W_g(\omega)\mathrm{d}\omega = 0$, because the frequency weighting function $W_g(\omega)$ is an even function of frequency and hence $\omega W_g(\omega)$ is an odd function. Eq. (9) can thus be simplified to

$$\frac{\partial \hat{R}^g_{x_1x_2}(\tau)}{\partial\tau} \approx -\frac{1}{2\pi}\int_{-\infty}^{+\infty} \omega W_g(\omega)[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]\,\mathrm{d}\omega. \tag{10}$$

When $\tau = \hat{\tau}_{\mathrm{peak}}$, $(\partial \hat{R}^g_{x_1x_2}(\hat{\tau}_{\mathrm{peak}}))/\partial\tau = 0$, and since $\omega W_g(\omega)[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau]$ is an even function (product of two odd functions), Eq. (10) can be set to zero and written as

$$\int_0^{+\infty} \omega W_g(\omega)[\hat{\Phi}_{x_1x_2}(\omega)+\omega\tau_{\mathrm{peak}}]\mathrm{d}\omega = 0. \tag{11}$$

Expressing this as a summation with limits of $i = 1,2,\dots,N/2$ and frequency increment $\delta\omega$ gives

$$\sum_{i=1}^{N/2} W_i\hat{\Phi}_i\omega_i\delta\omega + \sum_{i=1}^{N/2} W_i\hat{\tau}_{\mathrm{peak}}\omega_i^2\delta\omega = 0. \tag{12}$$

Rearranging Eq. (12) gives the time delay estimate as

$$\hat{\tau}_{\mathrm{peak}} = -\frac{\sum_{i=1}^{N/2} W_i\hat{\Phi}_i\omega_i}{\sum_{i=1}^{N/2} W_i\omega_i^2}, \tag{13}$$

which is identical to Eq. (5), the equation for the GPS method. In the derivation of Eq. (13), which started with the definition of the weighted cross-correlation function given in Eq. (2), it is important to note the following conditions:

(a) The relationship between the weighting function for the phase spectrum W and the weighting function Ψ applied in the GCC methods given in Eq. (7).

(b) The requirement for a small deviation of the estimate of the phase spectrum from the actual phase spectrum at every frequency in the range of interest.

If $\Psi = 1$, i.e., corresponding to the BCC function, then combining Eqs. (7) and (13) gives

$$\hat{\tau}_{\text{peak}} = -\frac{\sum_{i=1}^{N/2} |\hat{S}_i| \hat{\Phi}_i \omega_i}{\sum_{i=1}^{N/2} |\hat{S}_i| \omega_i^2}. \tag{14}$$

Eq. (14) allows an interesting interpretation on how time delay estimation is achieved with the commonly used BCC function; the authors do not believe this can be found in the literature. It shows that the BCC effectively minimises the weighted mean square error between the estimated phase and the actual phase corresponding to the true time delay, where the weighting function is the modulus of the CSD.

Given a choice of weighting functions, it is not clear that the modulus of the CSD would be the weighting function of choice especially as it is influenced by the leak spectrum, wave propagation along the pipe and the frequency response of the sensors [8]. The information in the CSD related to the time delay is contained in the phase spectrum, thus a sensible weighting function would be to set $W = 1$, which is equivalent to whitening the modulus or the PHAT estimator. This is a reasonable weighting function if the phase data is of good quality *at each frequency*. If it is not then the least square fit may be unduly influenced by data of poor quality. The established weighting functions discussed previously in the literature are the PHAT, SCOT and ML functions and these are given in Table 1 for the GPS method. The SCOT and ML functions contain an estimate of the coherence function given by

$$\hat{\gamma}^2_{x_1x_2}(\omega) = \frac{|\hat{S}_{x_1x_2}(\omega)|^2}{\hat{S}_{x_1x_1}(\omega)\hat{S}_{x_2x_2}(\omega)}, \tag{15}$$

where $\hat{S}_{x_1x_1}(\omega)$ and $\hat{S}_{x_2x_2}(\omega)$ are the smooth estimates of the auto-spectral density functions $S_{x_1x_1}(\omega)$ and $S_{x_2x_2}(\omega)$, respectively. An intuitive development would be to use a weighting function in the GPS method which is simply the coherence so that the time delay is given by

$$\hat{\tau}_{\text{peak}} = -\frac{\sum_{i=1}^{N/2} \hat{\gamma}_i^2 \hat{\Phi}_i \omega_i}{\sum_{i=1}^{N/2} \hat{\gamma}_i^2 \omega_i^2}, \tag{16}$$

which is termed here the GPS-COH method. Thus the coherence function is used to suppress regions of the phase spectrum where poor coherence of the signals occurs. In practice there are various situations where the signal to noise ratio is very low at certain frequencies, or the assumption of uncorrelated background noise is violated. In such cases, the deviation of phase spectrum estimate $\hat{\Phi}_i$ from the actual value can be large. Hence, the error in the time delay estimate provided by the regression analysis of the phase spectrum can be large if the noise in the phase data is not taken into account. To overcome this, the coherence between the two sensor signals can be used to derive a more accurate time delay estimate.

The equivalent weighting function for the GCC-COH method is $\gamma^2_{x_1x_2}(\omega)/|S_{x_1x_2}(\omega)|$ which is given in Table 1. Of course the coherence function could be raised to different powers, and indeed the GPS-SCOT estimator has a weighting function which is simply the square root of the coherence.

The relationship between the BCC and the GCC methods including the PHAT, SCOT, ML and COH methods, with the corresponding GPS methods, is illustrated in Table 1. It should be noted that the GPS methods do not suffer from the disadvantage of the resolution problem that occurs in the GCC methods.

3.2. Comparison of the variances of the time delay estimates

Provided that the time delay estimate $\hat{\tau}_{\text{peak}}$ is in the neighbourhood of the true time delay and there are no other biasing effects, an expression for the variance of the time delay estimate calculated using the GCC methods is given by [3]

$$\sigma^2_{\hat{\tau}_{\mathrm{peak}}} = \frac{2\pi}{T} \frac{\int_{-\infty}^{\infty} \omega^2 |\Psi_g(\omega)|^2 \hat{S}_{x_1x_1}(\omega)\hat{S}_{x_2x_2}(\omega)[1-\hat{\gamma}^2_{x_1x_2}(\omega)]\,\mathrm{d}\omega}{[\int_{-\infty}^{\infty} \omega^2 |\Psi_g(\omega)||\hat{S}_{x_1x_2}(\omega)|\,\mathrm{d}\omega]^2}. \tag{17}$$

An equivalent expression for the time delay estimate can be derived from the variance of the phase estimate. For the case of small errors and assuming that the signals are stationary, the phase spectrum estimate given by the GPS method has a variance of approximately [6]

$$\sigma^2_{\hat{\Phi}_i} \approx \frac{1-\hat{\gamma}_i^2}{2r\gamma_i^2}, \tag{18}$$

where $\hat{\gamma}_i^2$ denotes the discrete ordinary coherence function estimate and r is the number of data segments such that rN is the number of data points in the complete time history. The variance, $\sigma^2_{\hat{\tau}_{\mathrm{peak}}}$, of the best estimate of the time delay is given by

$$\begin{aligned}\sigma^2_{\hat{\tau}_{\mathrm{peak}}} &= \frac{|W_1|^2\omega_1^2\sigma^2_{\hat{\Phi}1} + |W_2|^2\omega_2^2\sigma^2_{\hat{\Phi}2} + \cdots + |W_{N/2}|^2\omega^2_{N/2}\sigma^2_{\hat{\Phi}_{N/2}}}{\left(\sum_{i=1}^{N/2}|W_i|\omega_i^2\right)^2} \\ &= \frac{\sum_{i=1}^{N/2}|W_i|^2\omega_i^2\sigma^2_{\hat{\Phi}i}}{\left(\sum_{i=1}^{N/2}|W_i|\omega_i^2\right)^2}.\end{aligned} \tag{19}$$

Substituting Eqs. (18) into (19) gives

$$\sigma^2_{\hat{\tau}_{\mathrm{peak}}} = \frac{\sum_{i=1}^{N/2}|W_i|^2\omega_i^2((1-\hat{\gamma}_i^2)/\hat{\gamma}_i^2)}{2r\left(\sum_{i=1}^{N/2}|W_i|\omega_i^2\right)^2}, \tag{20}$$

which shows that poor coherence at some frequencies will cause uncertainty in the time delay estimate.

To compare the variances for the GCC and GPS methods, the variance expression for the GCC method written in integral form in Eq. (17) is converted to discrete form. Combined with the definition of the coherence function given by Eq. (15), and noting the relationship between the weighting functions given in Eqs. (7), Eq. (17) can be written as

$$\sigma^2_{\hat{\tau}_{\mathrm{peak}}} = \frac{2\pi}{T} \frac{\int_{-\infty}^{\infty} \omega^2 |W_g(\omega)|^2 (1-\hat{\gamma}^2_{x_1x_2}(\omega))/(\hat{\gamma}^2_{x_1x_2}(\omega))\,\mathrm{d}\omega}{[\int_{-\infty}^{\infty} \omega^2 |W_g(\omega)|\mathrm{d}\omega]^2}. \tag{21}$$

The integrals in Eq. (21) can be written as summations, and because the functions in the numerator and denominator are even functions, the summations can be written with limits of $i = 1,2,\ldots N/2$ and frequency increment $\delta\omega$ to give

$$\sigma^2_{\hat{\tau}_{\mathrm{peak}}} = \frac{\pi}{T} \frac{\sum_{i=1}^{N/2}|W_i|^2\omega_i^2((1-\hat{\gamma}_i^2)/\hat{\gamma}_i^2)\delta\omega}{\left(\sum_{i=1}^{N/2}|W_i|\omega_i^2\delta\omega\right)^2}. \tag{22}$$

Now, $\delta\omega = 2\pi/T_r$ and the total length of the time history is related to the record length T_r by $T = rT_r$, so Eq. (22) can be written as

$$\sigma^2_{\hat{\tau}_{\mathrm{peak}}} = \frac{\sum_{i=1}^{N/2}|W_i|^2\omega_i^2((1-\hat{\gamma}_i^2)/\hat{\gamma}_i^2)}{2r\left(\sum_{i=1}^{N/2}|W_i|\omega_i^2\right)^2}, \tag{23}$$

which is identical to the expression for the variance given in Eq. (20) for GCC methods.

4. Experimental validation

Test data from a specially constructed leak-detection facility located at a National Research Council site in Canada was used to validate the theoretical work. The description of the test site and measurement procedures are detailed in Ref. [10]. Referring to Fig. 1, a joint leak signal was measured by accelerometers and hydrophones at two access points (fire hydrants), on either side of the location of a suspected leak. The distance d between the two sensor signals was 102.6 m, and the distance d_1 from the leak to sensor 1 was 73.5 m. The aim of the test was to estimate the time delay $\hat{\tau}_{\text{peak}}$ that corresponds to the difference in arrival times between the acoustic signals at each sensor.

The signals were each passed through an anti-aliasing filter with the cut-off frequency set at 200 Hz. Signals of 60-s duration were then sampled at a frequency of 500 samples/s. Spectral analysis was performed on the sampled data using a 1024-point FFT, applying a Hanning window and averaging the power spectra.

The magnitude and phase of the CSD and the coherence of the accelerometer measured signals are plotted in Figs. 3(a–c). It can be seen from the phase spectrum that most of the leak noise was between about 30 and 140 Hz. The coherence between the accelerometer-measured signals was very poor apart from the frequency

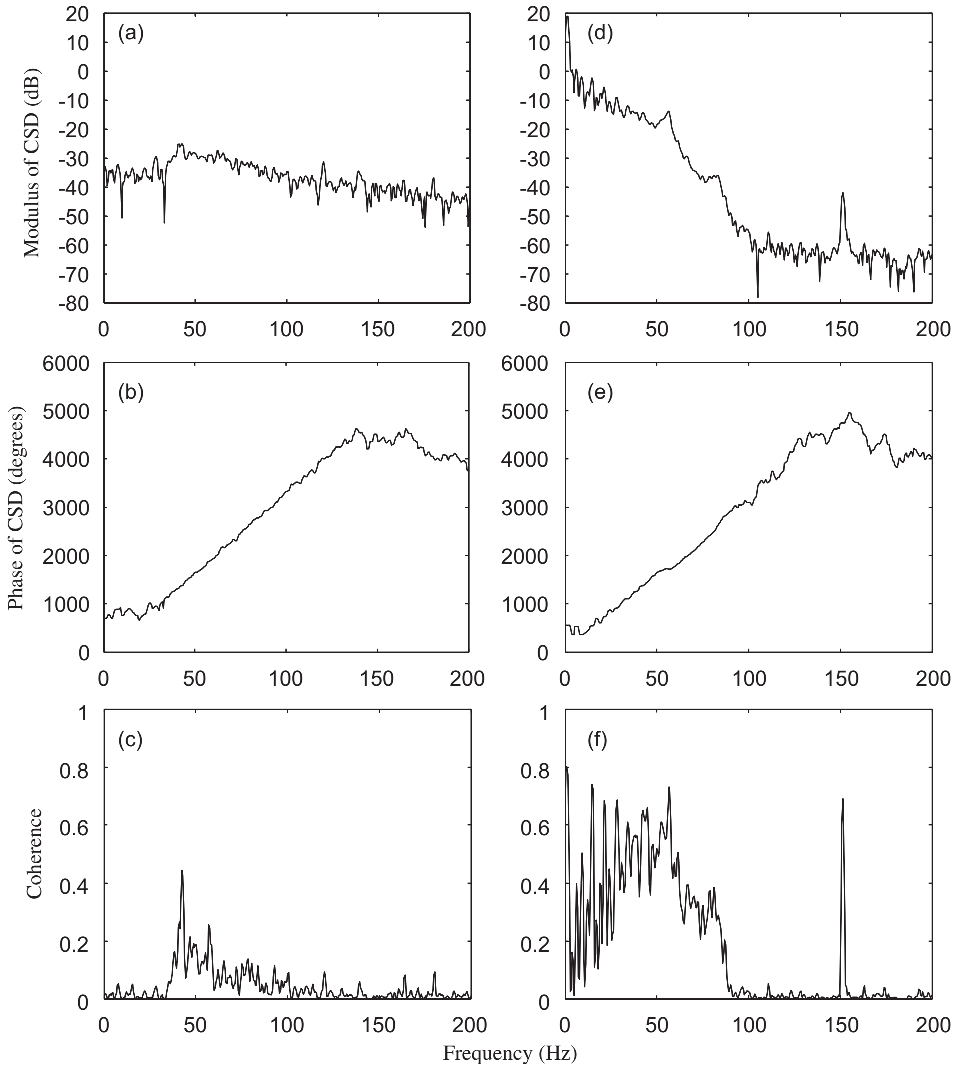

Fig. 3. The magnitude and phase of the CSD and the coherence of A1 and A2 signals measured with an accelerometer (a–c) and hydrophone (d–f). (a, d) modulus of the CSD (uncalibrated); (b, e) phase of the CSD (uncalibrated); (c, f) coherence function.

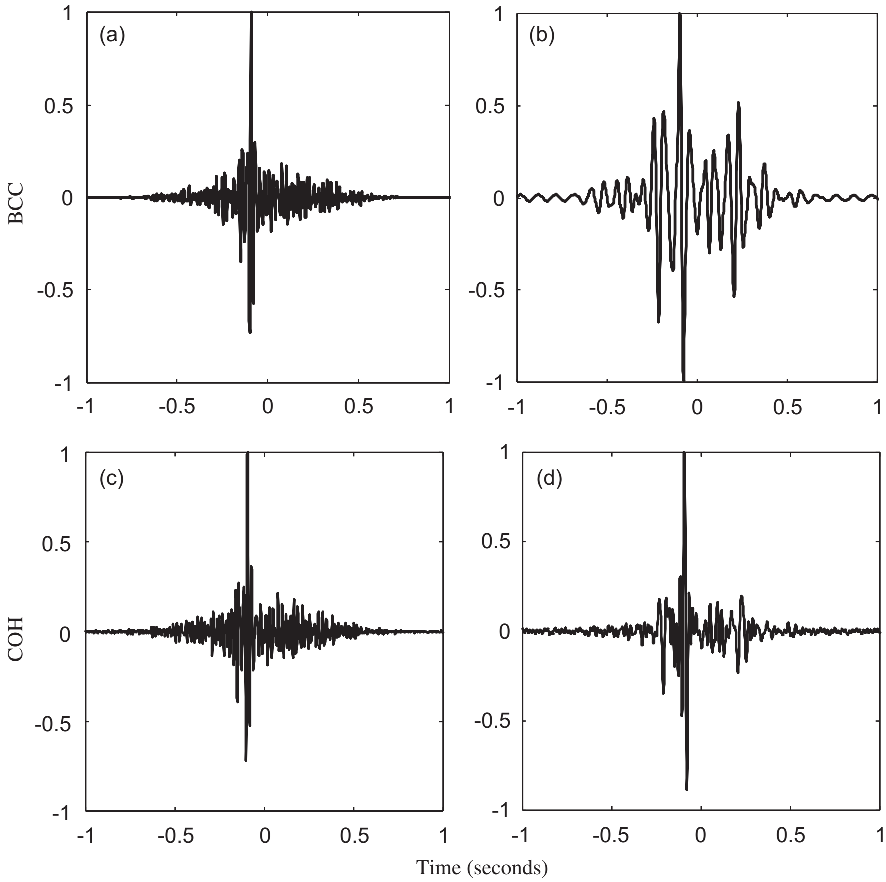

Fig. 4. Normalised cross-correlation of A1 and A2 accelerometer (a, c) and hydrophone (b, d) measured signals by using (a) BCC; (b) BCC; (c) COH; (d) COH estimators. The results are normalised to the peak correlation values.

band from about 40 to 100 Hz, as can be seen in Fig. 3(c). At low frequencies the signals are contaminated by ambient noise at low frequencies. At high frequencies the leak noise was very small due to the filtering effects of the pipe and thus ambient noise is again dominant. The magnitude and phase of the CSD and the coherence of the hydrophone measured signals are plotted in Figs. 3(d–f). It can be seen from the phase spectrum that most of the leak noise in this case was between about 10 and 120 Hz, and that the coherence was better than for the accelerometer signals in this bandwidth. However careful inspection of the phase spectrum shows that there was a spurious phase shift at about 60 Hz and again at about 80 Hz. If the phase is analysed over the whole range from 10 to 120 Hz an incorrect time delay will be predicted. It is thought that these spurious phase changes were due to hydrophone mounting resonances. Thus the usable frequency range to predict the time delay was taken to be 10–50 Hz.

The time delay was estimated using the BCC and GCC-COH methods, with weighting functions listed in Table 1. To compare the two correlation methods for time delay estimation, the results were normalised to the corresponding peak correlation values, and are plotted in Fig. 4(a–d). It can be seen that there is very little difference between the BCC and the GCC-COH methods for accelerometer signals. It can also be seen that the GCC-COH method gives a more distinct peak correlation with a narrower peak and smaller variance than the BCC method for hydrophone signals. The time delay was also calculated using the corresponding GPS-BCC and GPS-COH methods and the results are given in Table 2. It is evident that they give almost identical results to the GCC methods as expected. The discrepancy in the time delay estimate is due to the sampling frequency of 500 samples/second, which gives a time domain resolution of 0.002 s. The results also show that the random error in the time delay estimate is minimal for all the methods considered.

Table 2
Results of the time delay estimators using the GCC and GPS methods (band-pass filtering operations were applied)

		COH		BCC	
		GCC	GPS	GCC	GPS
Hydrophone-measured data	$\hat{\tau}_{\text{peak}}$ (s)	−0.0920	−0.0912	−0.0940	−0.0937
	$\left\lvert\frac{\sigma_{\hat{\tau}_{\text{peak}}}}{\hat{\tau}_{\text{peak}}}\right\rvert \times 100$	0.0725	0.0731	0.1200	0.1204
Accelerometer-measured data	$\hat{\tau}_{\text{peak}}$ (s)	−0.0900	−0.0910	−0.0900	−0.0909
	$\left\lvert\frac{\sigma_{\hat{\tau}_{\text{peak}}}}{\hat{\tau}_{\text{peak}}}\right\rvert \times 100$	0.0784	0.0758	0.0870	0.0861

Note: GPS methods are applied in the frequency range from 10 to 50 Hz for hydrophone-measured signals and from 30 to 140 Hz for accelerometer-measured signals.

5. Conclusions

In this paper, the equivalence between the time and frequency domain methods to estimate time delay has been shown explicitly, and the conditions under which both methods are identical have been identified. Given these conditions it has been shown that calculating time delay using the BCC method is equivalent to fitting a straight line to the phase spectrum and minimising the weighted mean square error between this line and the actual phase spectrum, where the weighting function is the modulus of the CSD. Rather than use this weighting function, it is possible to use other weighting functions and these are directly related to those used in the GCC methods.

To illustrate and compare the results from both GCC and GPS methods they have been applied to the application of finding a leak from buried water distribution pipes. It was demonstrated in this application that there is little difference between determining the time delay between the leak signals in the time domain using GCC methods or in the frequency domain using GPS methods. Test results show that both the time delay estimates and their variances using both methods are similar.

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Appendix. Generalised cross-correlation methods

In this appendix the generalised cross-correlation (GCC) methods used in this paper are described. They are based on the basic cross-correlation (BCC) method, but with some pre-filtering prior to performing the cross-correlation. Knapp and Carter [2] first discussed the characteristics of five GCC methods and compared them with the BCC method, however not all of the methods are appropriate for leak detection. The authors of this paper discussed a subset of the methods useful for this purpose [5], and these are outlined below.

Phase transform (PHAT): This filter removes the effect of the modulus of the cross-spectrum. It does this by "flattening" or "pre-whitening" the modulus of the cross-spectrum, leaving only the phase spectrum, which contains the information on the time delay between the two signals of interest. It has the desirable effect of sharpening the peak in the cross-correlation function. The deficiency in this technique is that it does not take into account the coherence between the signals and thus gives equal weight to all frequencies regardless of signal strength.

Smoothed coherence transform (SCOT). The SCOT estimator is similar to the PHAT estimator in that it "pre-whitens" the modulus of the cross-spectrum. Additionally, it also takes into account the effects of noise in the signals by multiplying the cross-spectrum by the square root of the coherence function. An alternative interpretation of this processor is given in this paper.

Maximum likelihood (ML). The ML estimator is similar to the SCOT estimator, but rather than multiplying the cross-spectrum by the square root of the coherence, $\gamma_{x_1x_2}(\omega)$, it is multiplied by $\gamma^2_{x_1x_2}(\omega)/[1-\gamma^2_{x_1x_2}(\omega)]$. The ML estimator gives the best estimate of the time delay, but not necessarily the sharpest and clearest peak in the cross-correlation function [5].

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